

Kaufman - Trading Systems & Methods - Chap02 14-23

Chap02 14-23

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The greater the standard deviation of returns, the greater the risk. In the securities industry, annual returns are most common, but monthly returns may be used if there are not enough years of data. There is no clear way to infer annual returns from monthly returns.

Downside Risk

Downside equity movements are often more important than profit patterns. It seems sensible that, if you want to know the probability of a loss, then you should study the history of equity drawdowns. The use of only the equity losses is called lower partial moments in which lower refers to the downside risk and partial means that only one side of the return distribution is used. A set of relative lower partial moments (RLPMs) is the expected value of the tracking error (equity drawdowns, the difference between the actual equity and the annualized returns) raised to the power of n :

$$\text{RLPM}_n = E[(R - B)^n], \quad \text{over the range where } R < B$$

$$= 0, \quad \text{over the range where } R \geq B$$

where R = return on investment

B = benchmark or corresponding regression return at that point in time

E = expected or mean return (described at the beginning of this section)

Therefore, the elements of the probability have only losses or zeros. The value n represents the order or ranking of the RLPMs. When $n = 0$, RLPM is the probability of a shortfall. Probability ($R < B$); when $n = 1$, RLPM is equal to the expected shortfall. $E[R - B]$ and when $n = 2$, RLPM is equal to the relative lower partial variance.

One concern about using only the drawdowns to predict other drawdowns is that it limits the number of cases and discards the likelihood that higher than normal profits can be related to higher overall risk. In situations where there are limited amounts of test data, both the gains and losses will offer needed information.

THE INDEX

The purpose of an average is to transform individuality into classification. When done properly, there is useful information to be gained. Indices have gained popularity in the futures markets recently; the stock market indices are now second to the financial markets in trading volume. These contracts allow both individual and institutional participants to invest in the overall market movement rather than take the higher risk of selecting individual securities. Furthermore, investors can hedge their current market position by taking a short position in the futures market against a long position in the stock market.

A less general index, the Dow Jones Industrials, or a grain or livestock index can help the trader take advantage of a more specific price without having to decide which products are more likely to do best. An index simplifies the decisionmaking process for trading. if an index does

not exist, it can be constructed to satisfy most purposes.

Constructing an Index

An index is traditionally used to determine relative value and normally expresses change as a percentage. Most indices have a starting value of 100 or 1,000 on a specific date. The index itself is a ratio of the current or composite values to those values during the base year. The selection of the base year is often chosen for convenience, but usually is far enough back to show a representative, stable price period. The base year for U.S. productivity and for unemployment is 1982, consumer confidence is 1985, and the composite of leading indicators is 1987. For example, for one market, the index for a specific year is

$$\text{Index (year } t) = \frac{\text{current price (year } t)}{\text{starting price (base year)}} \times 100$$

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If the value of the index is less than 100, the current value (year t) is lower than during the base year. The actual index value represents the percentage change.

For each year after the base year, the index value is the sum of the previous index value and the percentage change in price over the same period,

$$\text{current index value} = \text{previous index value} + \frac{\text{current price} - \text{previous price}}{\text{previous price}} \times 100$$

It is very convenient to create an index for two markets that trade in different units because they cannot be otherwise compared. For example, if you wanted to show the spread between gold and the U.S. Dollar Index, you could index them both beginning at the same date. The new indices would both be in the same units, percent, and would be easy to compare.

Most often, an index combines a number of related markets into a single number. A simple aggregate index is the ratio of unweighted sums of market prices in a specific year to the same markets in the base year. Most of the popular indices, such as the New York Stock Exchange Composite Index, fall into this class. A weighted aggregate index biases certain markets by weighting them to increase or decrease their effect on the composite value. The index is then calculated as in the simple aggregate index. When combining markets into a single index value, the total of all the weighting normally totals to the value one, although you may also divide the composite value by the total of all the individual weights.

U.S. Dollar Index

A practical example of a weighted index is the U.S. Dollar Index, traded on the New York Futures Exchange. In order of greatest weighting, the 10 currency components are the Deutschmark 20.8%, Japanese yen 13.6%, French franc 13.1%, British pound 11.9%, Canadian dollar 9.1%, Italian lira 9.0%, Netherlands guilder 8.3%, Belgian franc 6.4%, Swedish kroner 4.2%, and the Swiss franc 3.6%. This puts a total weight of 75.5% in European currencies with only the Japanese yen representing Asia, not a

practical mix for a world economy that has become dependent on Far Eastern trade. Within Europe, however, allocations seem to be proportional to the relative size of the economies.

The Dollar Index rises when the U.S. dollar rises. Quotes are in foreign exchange notation, where there are 1.25 Swiss francs per U.S. dollar, instead of .80 dollars per franc as quoted on the Chicago Mercantile Exchange's IMM. For example, when the Swiss franc moves from 1.25 to 1.30 per dollar, there are more Swiss francs per dollar; therefore, each Swiss franc is worth less.

In the daily calculation of the Dollar Index, each price change is represented as a percent. If, in our previous example, the Swiss franc rises .05 points, the change is $\frac{5}{125} = 0.04$; this is multiplied by its weighting factor .208 and contributes +.00832 to the Index.

PROBABILITY

Calculation must measure the incalculable.

Dixon G. Watts

Change is a term that causes great anxiety. However, the effects and likelihood of a chance occurrence can be measured, although not predicted. The area of study that deals with uncertainty is probability. Everyone uses probability in daily thinking and actions. When you tell someone that you will be there in 30 minutes, you are assuming:

Your car will start.

You will not have a breakdown.

You will have no unnecessary delays.

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You will drive at a predictable speed.

You will have the normal number of green lights.

All these circumstances are extremely probabilistic, and yet everyone makes the same assumptions. Actually, the 30-minute arrival is intended only as an estimate of the average time it should take for the trip. If the arrival time were critical, you would extend your estimate to 40 or 45 minutes, to account for unexpected events. In statistics, this is called increasing the confidence interval. You would not raise the time to 2 hours, because the likelihood of such a delay would be too remote. Estimates imply an allowable variation, all of which is considered normal.

Probability is the measuring of the uncertainty surrounding an average value. Probabilities are measured in percent of likelihood. For example, if M numbers from a total of N are expected to fall within a specific range, the probability P of any one number satisfying the criteria is

$$P = \frac{M}{N}, \quad 0 < P < 1$$

When making a trade, or forecasting prices, we can only talk in terms of probabilities or ranges. We expect prices to rise 30 to 40 points, or we have a 65% chance of a \$400 profit from a trade. Nothing is certain, but a high probability of success is very attractive.

Laws of Probability

Two basic principles in probability are easily explained by using

examples with playing cards. In a deck of 52 cards, there are 4 suits of 13 cards each. The probability of drawing a specific card on any one turn is 1/52. Similarly, the chances of drawing a particular suit or card number are 1/4 and 1/13, respectively. The probability of any one of these three possibilities occurring is the sum of their individual probabilities. This is known as the law of addition. The probability of success in choosing a numbered card, suit, or specific card is

$$P = \frac{1}{13} + \frac{1}{4} + \frac{1}{52} = \frac{18}{52} = 35\%$$

Another basic principle, the law of multiplications, states that the probability of two occurrences happening simultaneously or in succession is equal to the product of their separate probabilities. The likelihood of drawing a three and a club from the same deck in two consecutive turns (replacing the card after each draw) or of drawing the same cards from two decks simultaneously is

$$P = \frac{1}{13} \times \frac{1}{4} = \frac{1}{52} = 2\%$$

Joint and Marginal Probability

Price movement is not as clearly defined as a deck of cards. There is often a relationship between successive events. For example, over two consecutive days, prices must have one of the following sequences or joint events.. (up, up), (down, down), (up, down), (down, up), with the joint probabilities of .40, .10, .35, and .15, respectively in this example, there is the greatest expectation that prices will rise. The marginal probability of a price rise on the first day is shown in Table 22. Thus there is a 75% chance of higher prices on the first day and a 55% chance of higher prices on the second day.

Contingent Probability

What is the probability of an outcome contingent on the result of a prior event? In the example of joint probability, this might be the chance of a price increase on the second day

TABLE 2-2 Marginal Probability

		Day 2				Marginal Probability on Day 1
		Up	Down			
Day 1	Up	.40	.35	.75	Up	
	Down	.15	.10	.25	Down	
		.55	.45			
		Up	Down			
		Marginal Probability on Day 2				

when prices declined on the first day. The notation for this situation (the probability of A conditioned on B) is

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{\text{joint probability of A and B}}{\text{marginal probability of B}}$$

then

$$\begin{aligned} P(\text{up Day 2} | \text{down Day 1}) &= \frac{\text{joint probability of up Day 2 and down Day 1}}{\text{marginal probability of down Day 1}} \\ &= \frac{.15}{.25} = .60 \end{aligned}$$

The probability of either a price increase on Day 1 or Day 2 is

$$\begin{aligned} P(\text{either}) &= P(\text{up Day 1}) + P(\text{up Day 2}) - P(\text{up Day 1 and 2}) \\ &= .75 + .55 - .40 \\ &= .90 \end{aligned}$$

Markov Chains

If we believe that today's price movement is based in some part on what happened yesterday, we have a situation called conditional probability. This can be expressed as a Markov process, or Markov chain. The results, or outcomes, of a Markov chain express the probability of a state or condition occurring. For example, the possibility of a clear, cloudy, or rainy day tomorrow might be related to today's weather.

The different combinations of dependent possibilities are given by a transition matrix. In our weather prediction example, a clear day has a 70% chance of being followed by another clear day, a 25% chance of a cloudy day, and only a 5% chance of rain. In Table 23, each possibility today is shown on the left, and its probability of changing tomorrow is indicated across the top. Each row totals 100%, accounting for all weather combinations. The relationship between these events can be shown as a continuous network (see Figure 26).

The Markov process can reduce intricate relationships to a simpler form. First, consider a twostate process. Using the markets as an example, what is the probability of an up or down day following an up day, or following a down day? If there is a 70% chance of a higher day following a higher day and a 55% chance of a higher day following a lower day, what is the probability of any day within an uptrend being up?

TABLE 2-3 Transition Matrix

		Tomorrow		
		Clear	Cloudy	Rainy
Today	Clear	.70	.25	.05
	Cloudy	.20	.60	.20
	Rainy	.20	.40	.40

Start with either an up or down day, and then calculate the probability of the next day being up or down. This is done easily by simply counting the number of cases, given in Table 24a, then dividing to get the percentages, as shown in Table 24b.

Because the first day may be designated arbitrarily as up or down, it is an exception to the general rule and, therefore, is given the weight of 50%. The probability of the second day being up or down is the sum of the

joint probabilities

$$\begin{aligned} P(UP)_2 &= (.50 \times .70) + (.50 \times .55) \\ &= .625 \end{aligned}$$

The probability of the second day being up is 62.5%
the probability of an up day as .625, the down as .375

$$\begin{aligned} P(UP)_3 &= (.625 \times .70) + (.375 \times .55) \\ &= .64375 \end{aligned}$$

and the fourth day,

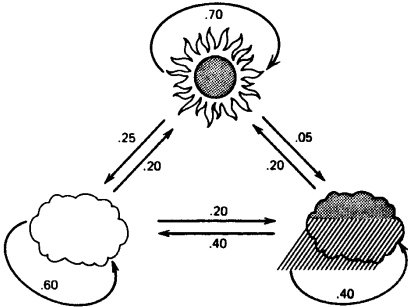
$$\begin{aligned} P(UP)_4 &= (.64375 \times .70) + (.35625 \times .55) \\ &= .64656 \end{aligned}$$

which can now be seen to be converging. To generalize
at what happens on the *i*th day:

$$P(up)_{i+1} = [P(up)_i \times .70] + [(1 - P(up)_i) \times .55]$$

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FIGURE 2-6 Probability network.



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TABLE 2-4a Counting the Occurrences of Up and Down Days

		Today		
		Up	Down	Total
Previous day	Up	75	60	135
	Down	60	65	125

TABLE 2-4b Starting Transition Matrix

		Today		
		Up	Down	Total
Previous day	Up	.555	.444	1.00
	Down	.480	.520	1.00

Because the probability is converging, the relationship

$$P(\text{up})_{i+1} = P(\text{up})_i$$

can be substituted and used to solve the equation

$$P(\text{up})_i = [P(\text{up})_i \times .70] + [.55 - P(\text{up})_i \times .55]$$

giving the probability of any day being up within an uptrend as

$$P(\text{up})_i = .64705$$

We can find the chance of an up or down day if the 5day trend is up simply by substituting the direction of the 5day trend (or nday trend) for the previous day's direction in the example just given.

Predicting the weather is a more involved case of multiple situations converging and may be very representative of the way prices react to past prices. By approaching the problem in the same manner as the twostate process, a 1/3 probability is assigned to each situation for the first day; the second day's probability is

$$\begin{aligned} P(\text{clear})_2 &= (.333 \times .70) + (.333 \times .20) + (.333 \times .20) \\ &= .3663 \end{aligned}$$

$$\begin{aligned} P(\text{cloudy})_2 &= (.333 \times .25) + (.333 \times .60) + (.333 \times .40) \\ &= .41625 \end{aligned}$$

$$\begin{aligned} P(\text{rainy})_2 &= (.333 \times .05) + (.333 \times .20) + (.333 \times .40) \\ &= .21645 \end{aligned}$$

Then using the second-day results, the third day is

$$\begin{aligned} P(\text{clear})_3 &= (.3663 \times .70) + (.41625 \times .20) + (.21645 \times .20) \\ &= .38295 \end{aligned}$$

$$\begin{aligned} P(\text{cloudy})_3 &= (.3663 \times .25) + (.41625 \times .60) + (.21645 \times .40) \\ &= .42791 \end{aligned}$$

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$$\begin{aligned} P(\text{rainy})_3 &= (.3663 \times .05) + (.41625 \times .20) + (.21645 \times .40) \\ &= .18815 \end{aligned}$$

The general form for solving these three equations is

$$P(\text{clear})_{i+1} = [P(\text{clear})_i \times .70] + [P(\text{cloudy})_i \times .20] + [P(\text{rainy})_i \times .20]$$

$$P(\text{cloudy})_{i+1} = [P(\text{clear})_i \times .25] + [P(\text{cloudy})_i \times .60] + [P(\text{rainy})_i \times .40]$$

$$P(\text{rainy})_{i+1} = [P(\text{clear})_i \times .05] + [P(\text{cloudy})_i \times .20] + [P(\text{rainy})_i \times .40]$$

where each $i + 1$ element can be set equal to the corresponding i th values; there are then three equations in three unknowns, which can be solved directly or by matrix multiplication, as shown in Appendix 3 ("Solution to Weather Probabilities Expressed as a Markov Chain"). Otherwise, it will be necessary to use the additional relationship

$$P(\text{clear})_i + P(\text{cloudy})_i + P(\text{rainy})_i = 1.00$$

The results are

$$P(\text{clear}) = .400$$

$$P(\text{cloudy}) = .425$$

$$P(\text{rainy}) = .175$$

BayesTheorem

Although historic generalization exists concerning the outcome of an event,

a specific current market situation may alter the probabilities. Bayes theorem combines the original probability estimates with the added event probability (the reliability of the new information) to get a posterior or revised probability,

$P(\text{original and added event})$

$P(\text{added event})$

Assume that the price changes $P(\text{up})$ and $P(\text{down})$ are both original probabilities and that an added event probability, such as a crop report, inventory stocks, or money supply announcement, is expected to have an overriding effect on tomorrow's movement. Then

New probability $P(\text{up} \mid \text{added-event}) =$

$\frac{P(\text{up and added-event})}{P(\text{up and added-event}) + P(\text{down and added-event})}$

where *up* and *down* refer to the original historic probability.

Bayes theorem finds the conditional probability even when the probabilities are not known.

New probability $P(\text{up} \mid \text{added-event}) =$

$\frac{P(\text{up}) \times P(\text{added-event} \mid \text{up})}{P(\text{up}) \times P(\text{added-event} \mid \text{up}) + P(\text{down}) \times P(\text{added-event} \mid \text{down})}$

where $P(\text{added-event} \mid \text{up})$ is the reliability of the new information for an upwards move, and $P(\text{added-event} \mid \text{down})$ is the reliability for a downwards move.

² A full mathematical treatment of Markov chains can be found in *Markov Chains* (Springer-Verlag, New York, 1976).

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when the added news indicates up. For example, if a decline in soybean planting by more than 10% has a 90% chance of causing prices to move higher, then

$P(\text{added event up}) = .90$

and

$P(\text{added event down}) = .10$

would be used in Bayes theorem.

SUPPLY AND DEMAND

Price is the balancing point of supply and demand. To estimate the future price of any product or explain its historic patterns, it will be necessary to relate the factors of supply and demand and then adjust for inflation, technological improvement, and other indicators common to econometric analysis. The following sections briefly describe these factors.

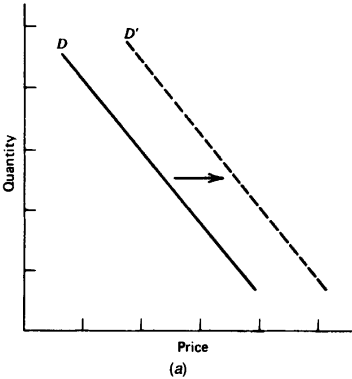
Demand

The demand for a product declines as price increases. The rate of decline is always dependent on the need for the product and its available substitutes at different price levels. In **Figure 27a**, D represents normal demand for a product over some fixed period. As prices rise, demand declines fairly rapidly. D' represents increased demand, resulting in higher prices at all levels.

Figure 27b represents the demand relationship for potatoes for the years 1929-1939. In most cases, the demand relationship is not a straight

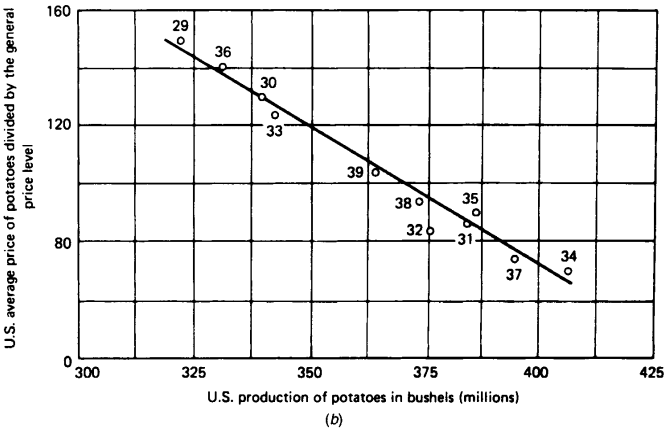
line; production costs and minimum demand prevent the price estimate from going to zero. On the higher end of the scale, there is a lag in the response to increased prices and a consumer reluctance to reduce purchasing even at higher prices (called inelastic demand). Figure 2-7c shows a

FIGURE 2-7 (a) Shift in demand. (b) Potatoes: U.S. average farm price on December 15th versus total production: 1929–1939. (c) Demand curve, including extremes.



Source: Shepherd, Geoffrey S., and G.A. Futrell. *Agricultural Price Analysis*. (Ames, IA: Iowa State University, 1969, p. 53.)

FIGURE 2-7 (Continued)

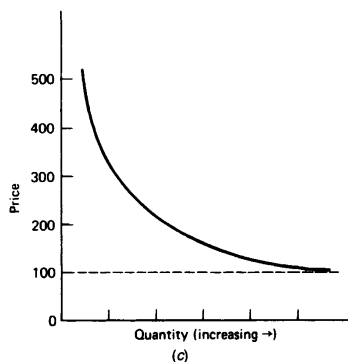


more representative demand curve, including extremes, where 100 represents the minimum acceptable income for a producer. The demand curve, therefore, shows the rate at which a change in quantity demanded brings about a change in price.

Elasticity of Demand

Elasticity is the key factor in expressing the relationship between price and demand. It is the relative change in demand as price increases,

FIGURE 2-7 (Continued)



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$$E_D = \frac{\text{relative change (\%) in demand}}{\text{relative change (\%) in price}}$$

A market that always consumes the same amount is called **inelastic**: as price rises, the demand remains small. An **elastic** market is just the opposite: as price falls, demand increases. E_D is negatively very large. Figure 2-8 shows demand elasticities.

If supply increases for a product that has existing consumer purchasing habits will require time to adjust. A shift from relatively inelastic (Figure 2-8b) to relatively elastic (Figure 2-8c) would be a significant shift.

Supply

The supply side of the economic equation is the relationship between price and quantity supplied. Figure 2-9a shows that, as price increases, the supplier will increase the quantity supplied of the product. Figure 2-9b demonstrates the supply curve. When production costs are low, there is a nominal supply by the producer. When production costs are high, the supply is erratic. There may be insufficient supply to meet demand due to high fixed costs and difficulty restarting production. The appearance of new supplies or substitutes, as in Figure 2-9c, however, it is demand that brings price down.

Elasticity of Supply

The elasticity of supply E_S is the relationship between the relative change in price and the relative change in quantity supplied.

$$E_S = \frac{\text{relative change (\%) in supply}}{\text{relative change (\%) in price}}$$

The elasticity of supply, the counterpart of demand elasticity, is a positive number, because price and quantity move in the same direction at the same time.

Equilibrium

The demand for a product and the supply of that product meet at a point of equilibrium. The current price of any commodity, or any market, represents the point of equilibrium for that product at that moment in time. **Figure 210** shows a constant demand line D and a shifting supply, increasing to the right from S to S' .

FIGURE 2-8 Demand elasticity. (a) Relatively elastic. (b) Relatively inelastic. (c) Normal market.

